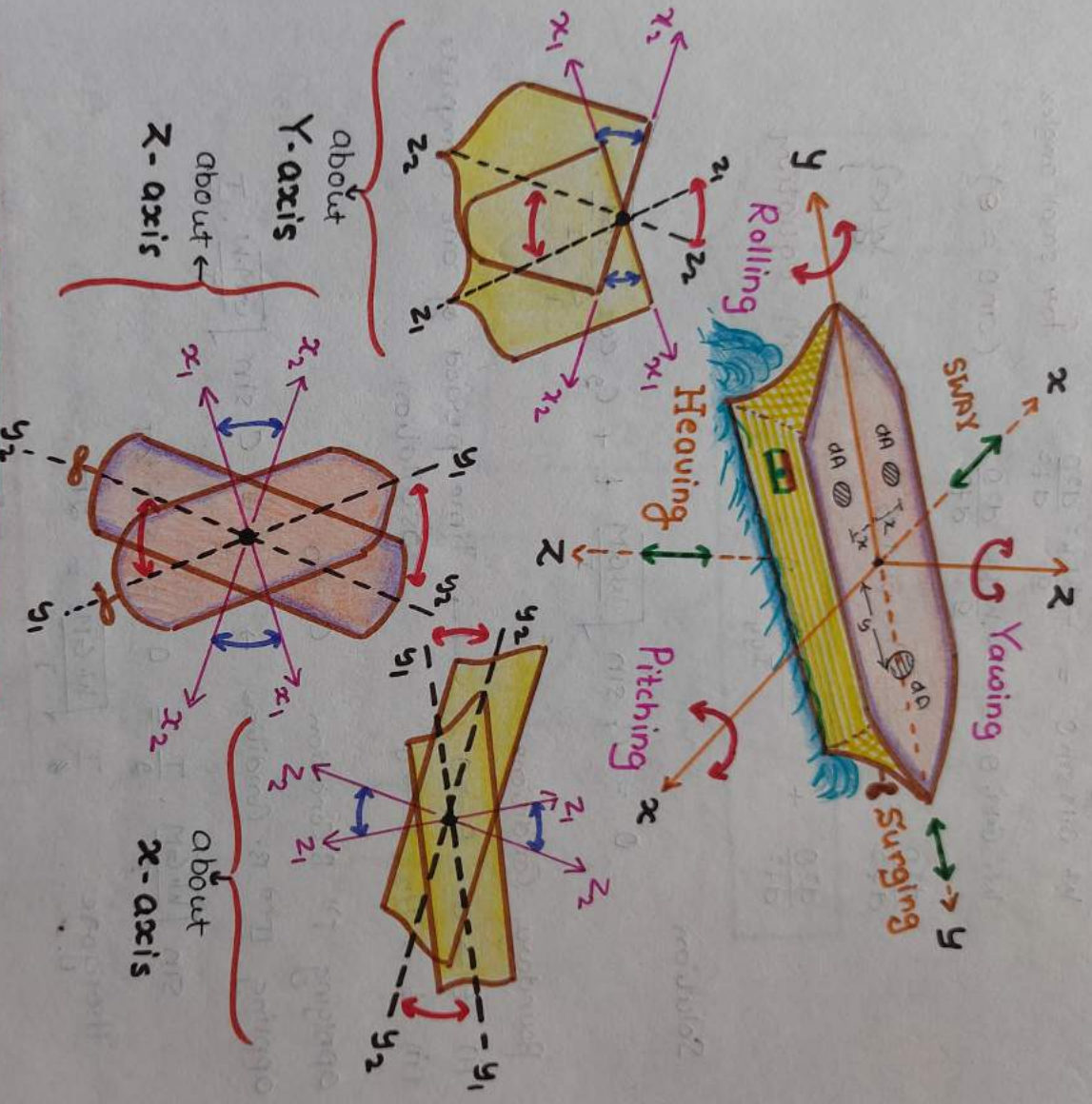


Ship Movements



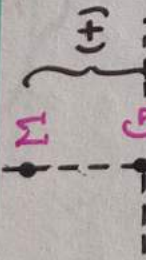
Because I Rolling < I Pitching

We should take very much care about Rolling because it is less, ship can roll very easily, therefore Management of Rolling very important and it automatically amends the Pitching also.

Equilibrium of Floating Bodies

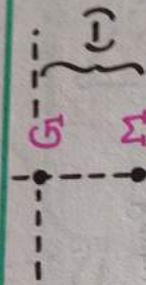
Equilibrium

STABLE EQUILLIBRIUM



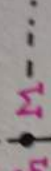
$$GM > 0$$

UNSTABLE EQUILLIBRIUM

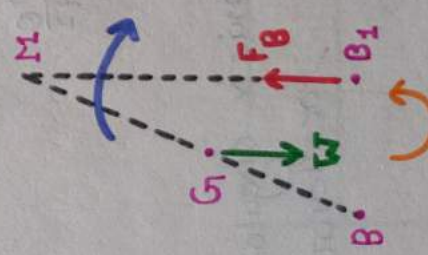
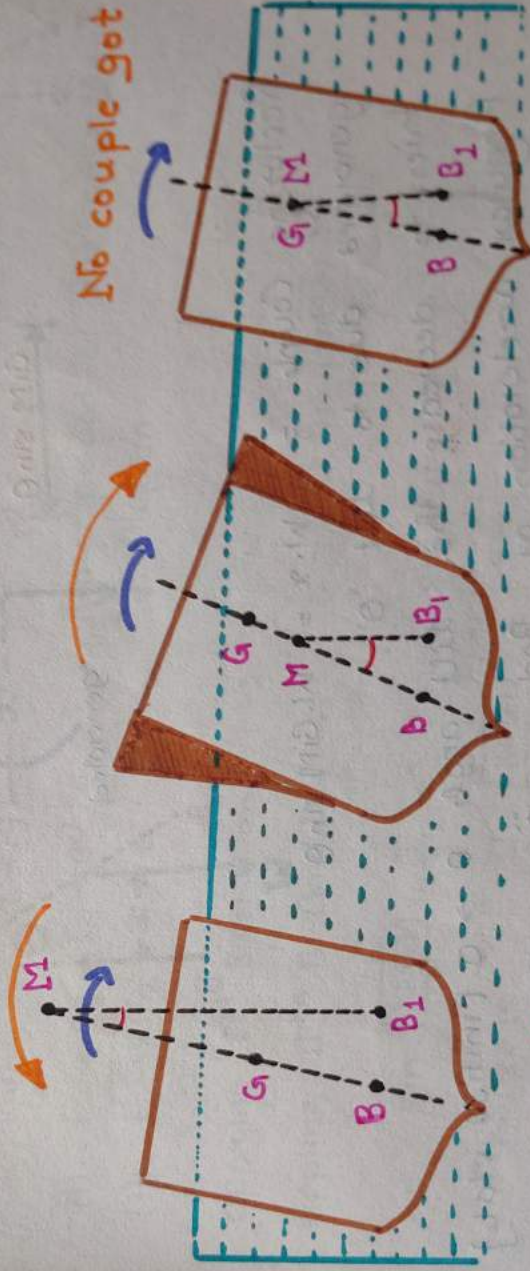


$$GM < 0$$

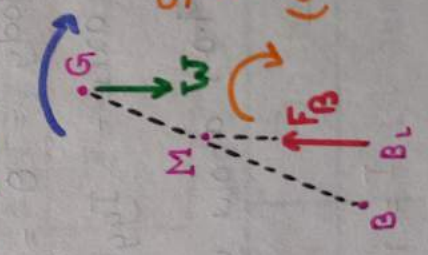
NEUTRAL EQUILLIBRIUM



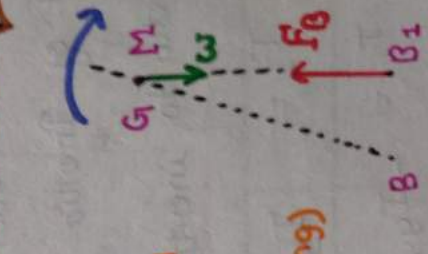
$$GM = 0$$



Restoring Couple



Generated Couple (disturbing)

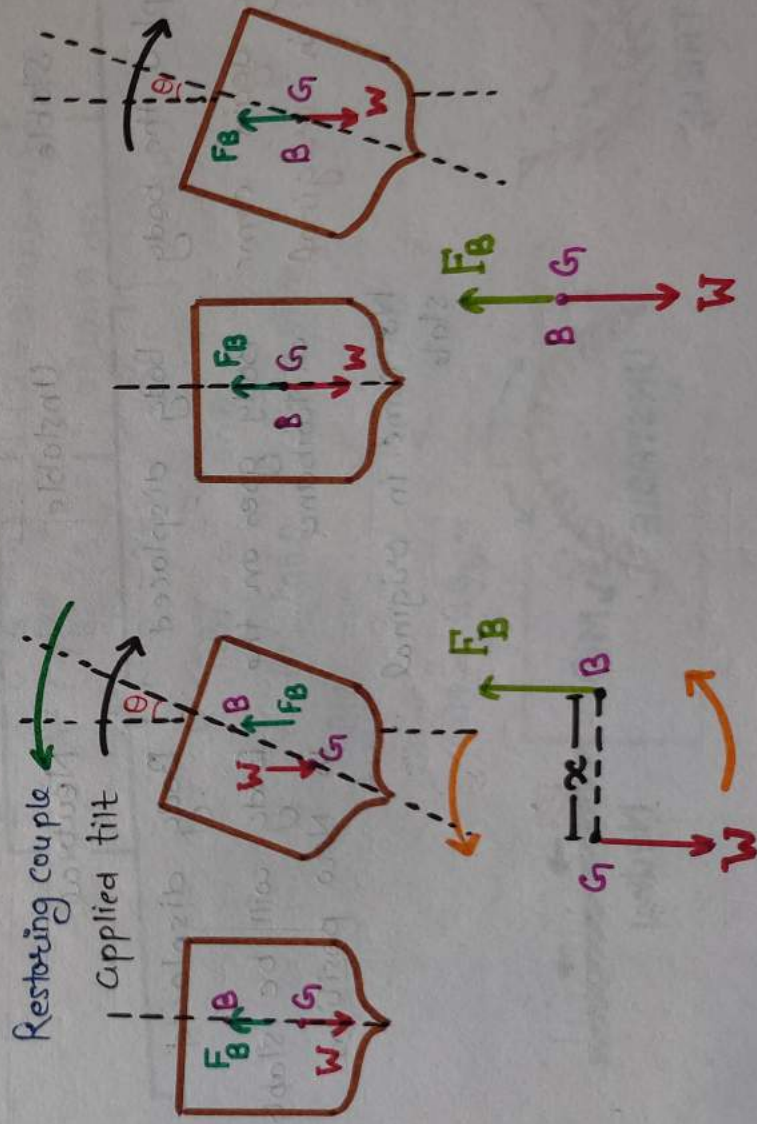
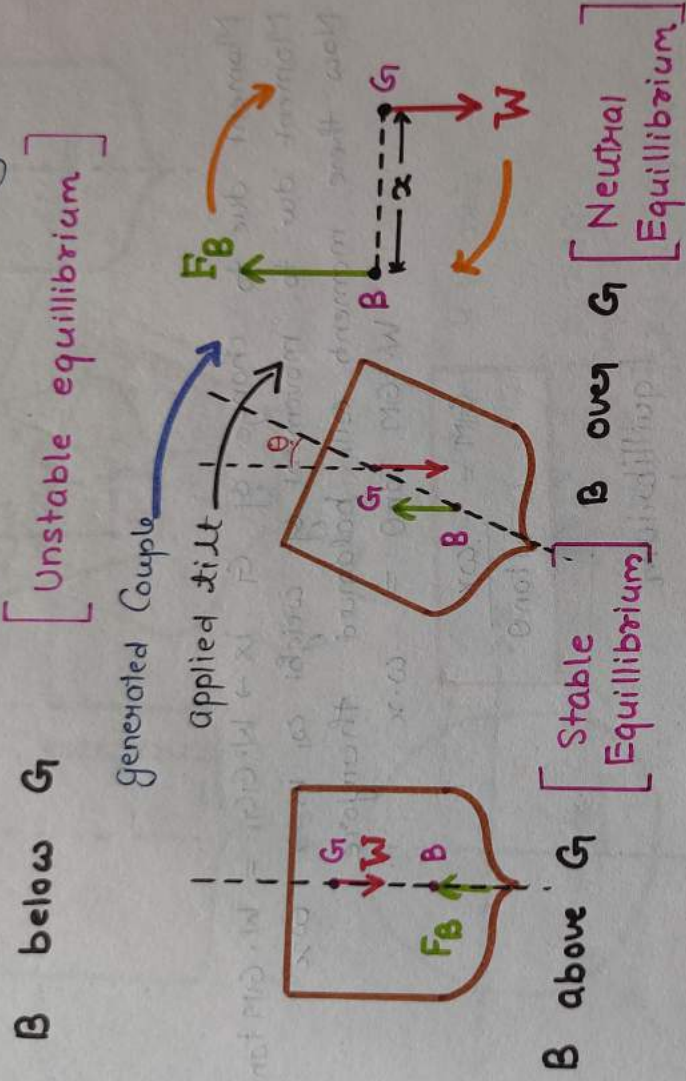


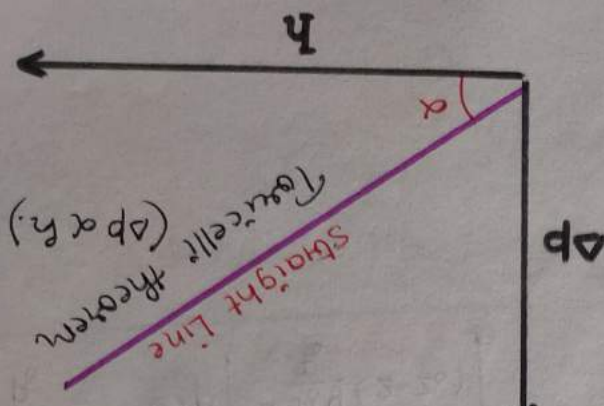
Balanced Each other (No couple)

$$\frac{G \cdot b}{G \cdot b} = \frac{M \cdot b}{M \cdot b}$$

Equilibrium of submerged unconstrained bodies

Unconstrained means \rightarrow Only weight and Buoyancy force (F_B) are acting.





$$\therefore (\alpha = \tan^{-1}(\rho g))$$

$$\tan \alpha = \rho g$$

$$\text{Slope} = \rho g$$

Linearly change with height

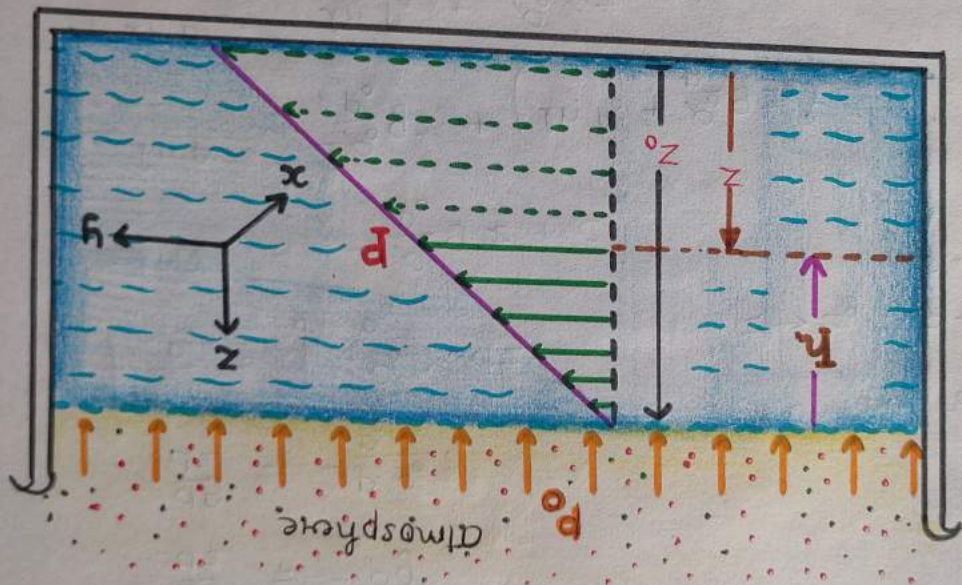
Pressure excess

$$\Delta p = \rho g h$$

depth from free surface

$$(p - p_0) = \rho g (z_0 - z)$$

Now



$$(p - p_0) = \rho g (z_0 - z)$$

then $p = -\rho g z + c$ and $p_0 = -\rho g z_0 + c$

when $z \rightarrow z_0 \Rightarrow p \rightarrow p_0 \rightarrow$ atmospheric pressure
 when $z \rightarrow z \Rightarrow p \rightarrow p$

establishing proper boundary condition.

Now Basic Equation of fluid at rest is

$$\nabla p = \rho \vec{b}$$

$$\left(i \frac{\partial p}{\partial x} + j \frac{\partial p}{\partial y} + k \frac{\partial p}{\partial z} = i \rho b_x + j \rho b_y + k \rho b_z \right)$$

Conclusion

$$\frac{\partial p}{\partial x} = \rho b_x$$

$$\frac{\partial p}{\partial y} = \rho b_y$$

$$\frac{\partial p}{\partial z} = \rho b_z$$

If gravity is the only body force acting downwards on body

$$b_x = 0 \rightarrow \frac{\partial p}{\partial x} = 0 \rightarrow p \neq p(x)$$

$$b_y = 0 \rightarrow \frac{\partial p}{\partial y} = 0 \rightarrow p \neq p(y) \rightarrow p \neq p(x, y)$$

$$b_z = -g \rightarrow \frac{\partial p}{\partial z} = -g \rightarrow p = p(z) \text{ only}$$

therefore

$$\frac{dp}{dz} = -\rho g$$

Ordinary differential Equation

Hydrostatic equation of fluid

Solutions of ODE

Incompressible fluid

Here we regard density $\rho = \text{constant}$

$$dp = -\rho g dz$$

$$\int dp = \int -\rho g dz$$

$$[p = -\rho g z + c]$$

Found by

Stable

Boundary condition

$\rho \neq \rho(x, y, z, t)$ generally

Δp very very low/small

→ Negligible

Now, the net surface force acting on the fluid element along i th direction is given by

$$\bar{F}_{\text{surface}, i} = \iint_{CS} T_i^j dA = \iint_{CS} (\tau_{ij} \cdot \hat{n}) dA$$

Divergence theorem $\iint_{CS} (\bar{F} \cdot \hat{n}) dA = \iiint_{CV} (\nabla \cdot \bar{F}) dV$

Now applying

$$\begin{aligned} \bar{F}_{\text{surface}, i} &\Rightarrow \iiint_{CV} (\nabla \cdot \bar{\tau}_{ij}) dV \\ \bar{F}_{\text{surface}, i} &\Rightarrow \iiint_{CV} \frac{\partial \tau_{ij}}{\partial x_j} dV \end{aligned}$$

If the fluid is at rest, only Normal component of stress on the surface, that are independent of the orientation of the surface,

stress tensor is Isotropic / Spherically symmetric

therefore $\tau_{ij} = -p \delta_{ij}$

thermodynamic pressure

$$\bar{F}_{\text{surface}, i} = \iiint_{CV} - \frac{\partial p}{\partial x_i} dV = \iiint_{CV} - \nabla p dV$$

Net surface force acting on the element

$$\bar{F}_{\text{Body}} = \iiint_{CV} \bar{b} \rho dV$$

Under equilibrium

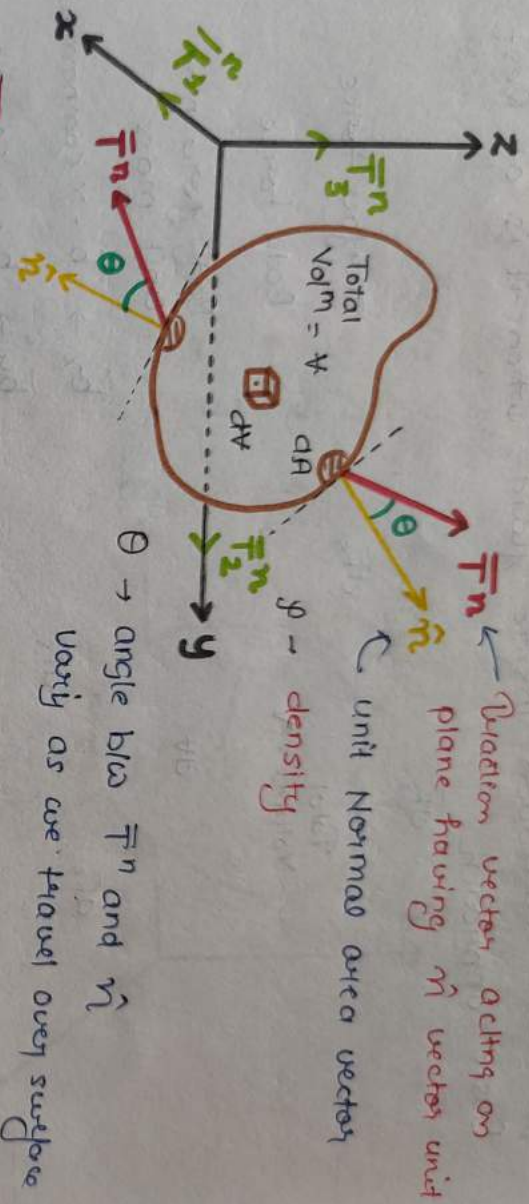
$$\bar{F}_{\text{surface}} + \bar{F}_{\text{Body}} = 0$$

$$\iiint_{CV} -\nabla p dV + \iiint_{CV} \bar{b} \rho dV = 0$$

$$\nabla p = \rho \bar{b}$$

Advanced approach

We will derive Basic equation using the concept of tensors [stress vector] basis.



\vec{F}_i^n = component of \vec{F}^n along \hat{i}

- 1 along x
- 2 along y
- 3 along z

We can represent the traction on any arbitrary plane with linear combination of its components of traction with respect to reference cartesian system

Cauchy theorem - $\vec{F}_i^n = \sum_{j=1}^3 \tau_{ji} \hat{n}_j$

from angular momentum conservation

Now $\vec{F}_i^n = \tau_{ij} \hat{n}_j$

Here

$\hat{n} = n_1 \hat{i} + n_2 \hat{j} + n_3 \hat{k}$

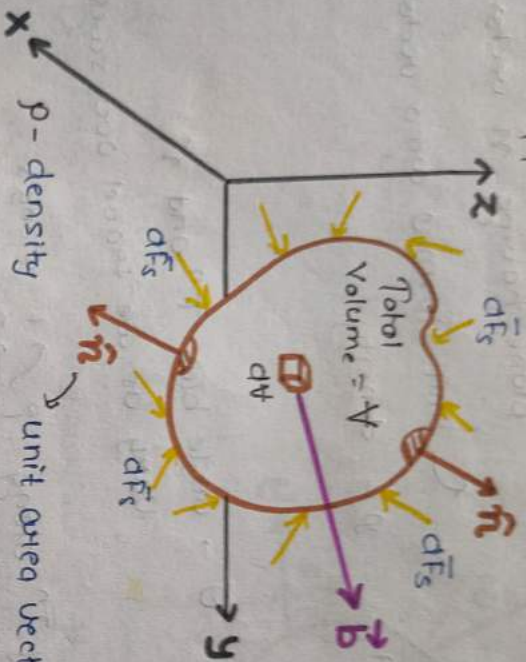
$\tau_{ij} n_j = \tau_{i1} n_1 + \tau_{i2} n_2 + \tau_{i3} n_3$
defining about

$\tau_i = \tau_{i1} \hat{i} + \tau_{i2} \hat{j} + \tau_{i3} \hat{k}$

$\tau_{ji} = \tau_{ij}$
Symmetric
Tensors

Basic Eqⁿ of Fluid statics.

We will derive fundamental eqⁿ that shows about pressure variation inside fluid when it is at rest
Basic approach



- $\vec{F}_s \rightarrow$ Surface force
- $\vec{F}_b \rightarrow$ Body force
- $\vec{b} \rightarrow$ Body force per unit mass
- $\vec{b}_x \rightarrow$ x component
- $\vec{b}_y \rightarrow$ y component
- $\vec{b}_z \rightarrow$ z component

Total Body force $\vec{F}_B = \iiint_{CV} \rho \vec{b} dV$

Total surface force $\vec{F}_S = \iint_{CA} -p \cdot d\vec{A} = \iint -p \hat{n} dA$

Body under equilibrium

Body force + Surface force = 0

$$\vec{F}_B + \vec{F}_S = 0$$

$$\iiint_V \rho \vec{b} dV + \iint_A -p \hat{n} dA = 0$$

$$\iiint_V \rho \vec{b} dV - \iiint_V \nabla p dV = 0$$

$$\iiint_V (\nabla p - \rho \vec{b}) dV = 0$$

$\nabla p = \rho \vec{b}$

(if F is scalar)

$$\left\{ \begin{aligned} \iint_A \vec{F} \cdot d\vec{A} &= \iiint_V (\nabla \cdot \vec{F}) dV \\ \iint_A (\vec{F} \cdot d\vec{A}) &= \iiint_V (\nabla \cdot \vec{F}) dV \end{aligned} \right.$$

Gauss-divergence theorem

We know $dV = \left(\frac{dx}{6} \frac{dy}{6} \frac{dz}{6} \right)$ of Rectangular element

if $\left. \begin{matrix} dx \rightarrow 0 \\ dy \rightarrow 0 \\ dz \rightarrow 0 \end{matrix} \right\} \rightarrow \frac{dx dy dz}{6} \rightarrow 0 \left\} dV \rightarrow 0 \right.$

Now $F_z = (\sigma_z - \sigma_n) dA_n \cos \gamma$ Equilibrium \downarrow

$F_x = 0$

$F_y = 0$

$F_z = 0$

$(\sigma_x - \sigma_n) dA_n \cos \alpha = 0$

$(\sigma_y - \sigma_n) dA_n \cos \beta = 0$

$(\sigma_z - \sigma_n) dA_n \cos \gamma = 0$

$\sigma_x = \sigma_n$

$\sigma_y = \sigma_n$

$\sigma_z = \sigma_n$

Pascal's Law

$\sigma_x = \sigma_y = \sigma_z = \sigma_n = -p$

$p \rightarrow$ thermodynamic pressure / OR / also called Hydrostatic pressure } compressive in Nature

Mean pressure, Scalar quantity

NOTE

$(\sigma_x = \sigma_y = \sigma_z = -p = k)$ as according to pascal's law

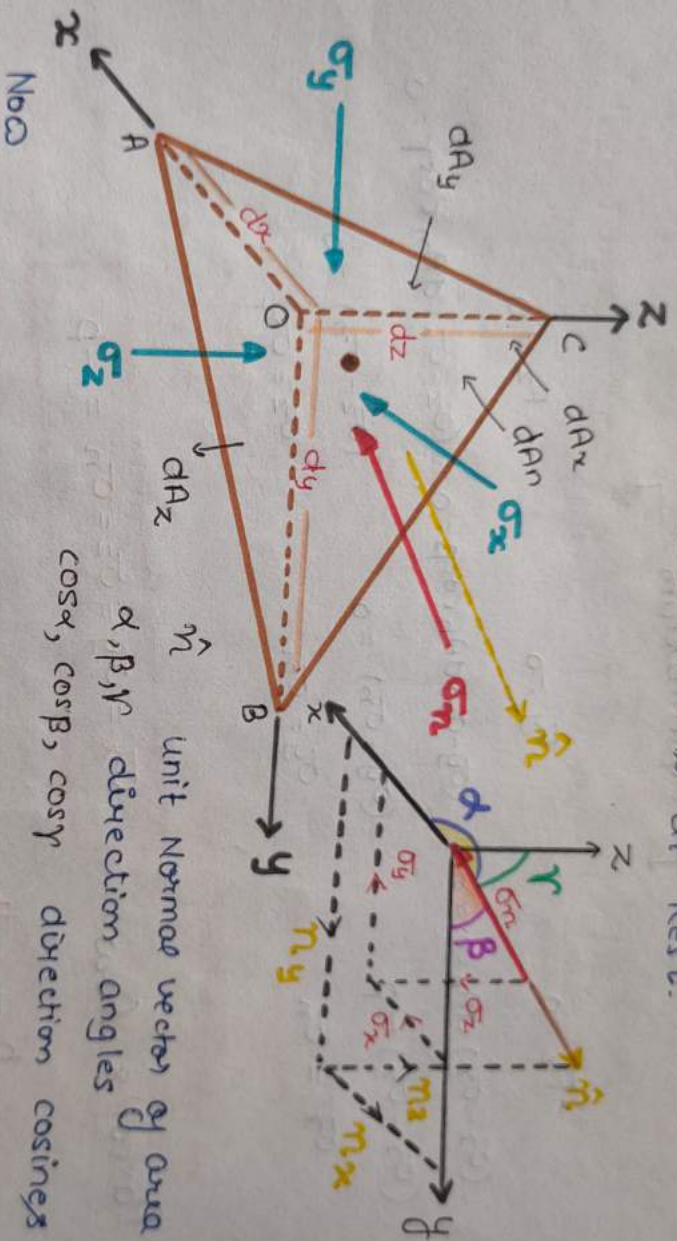
$\left. \begin{matrix} \sigma_x = k \\ \sigma_y = k \\ \sigma_z = k \end{matrix} \right\} \text{adding} \rightarrow \left. \begin{matrix} (\sigma_x + \sigma_y + \sigma_z) = 3k \\ (\sigma_x + \sigma_y + \sigma_z) = k \end{matrix} \right\}$

$p = - \frac{(\sigma_x + \sigma_y + \sigma_z)}{3} = - \frac{1}{3} [\sigma]$

PRESSURE and IT'S MEASUREMENT

Pascal's Law :: State of force in Rest fluid.

Now we have to analyse about the force inside the fluid domain when our fluid is at Rest.



\hat{n} unit Normal vector of area
 α, β, γ direction angles
 $\cos \alpha, \cos \beta, \cos \gamma$ direction cosines

$$F_x = \sigma_x \cdot \frac{dy dz}{2} - \sigma_x \Rightarrow \sigma_x dA_n \cos \alpha - \sigma_n dA_n \cos \alpha$$

$$F_y = \sigma_y \cdot \frac{dx dz}{2} - \sigma_y \Rightarrow \sigma_y dA_n \cos \beta - \sigma_n dA_n \cos \beta$$

$$F_z = \sigma_z \frac{dx dy}{2} - \sigma_n dA_n \cos \gamma - \rho dV \cdot g$$

Component of σ_n along -z direction
 Surface force

Body weight
 Body force

$$F_z = \sigma_z dA_n \cos \gamma - \sigma_n dA_n \cos \gamma - \rho \frac{dx dy dz}{6} \cdot g$$

Newton's Law of Viscosity

Stress \propto Rate of strain

$$\tau \propto \dot{\gamma}$$

$$\tau \propto \left(\frac{du}{dy}\right)$$

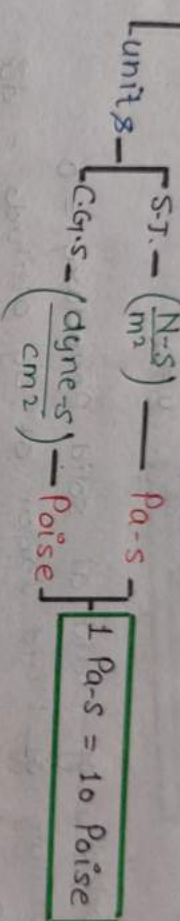
$$\left(\frac{du}{dy}\right)$$

Velocity gradient

$$\tau = \mu \left(\frac{du}{dy}\right)$$

$\mu \rightarrow$ viscosity coeff.

also $\mu \rightarrow$ dynamic viscosity of fluid



Fluid \rightarrow a substance which deform continuously under action of very small shear stress

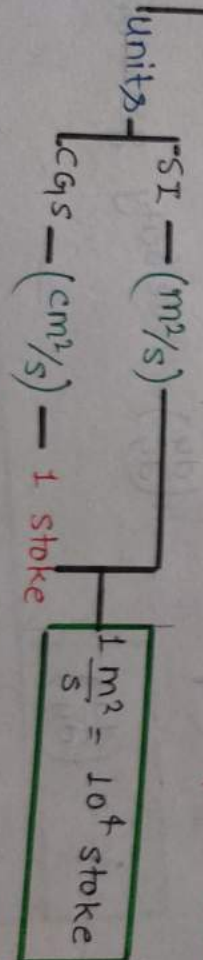
Also One more viscosity comes into existence

$\nu =$ kinematic viscosity

$$\nu = \frac{\mu}{\rho}$$

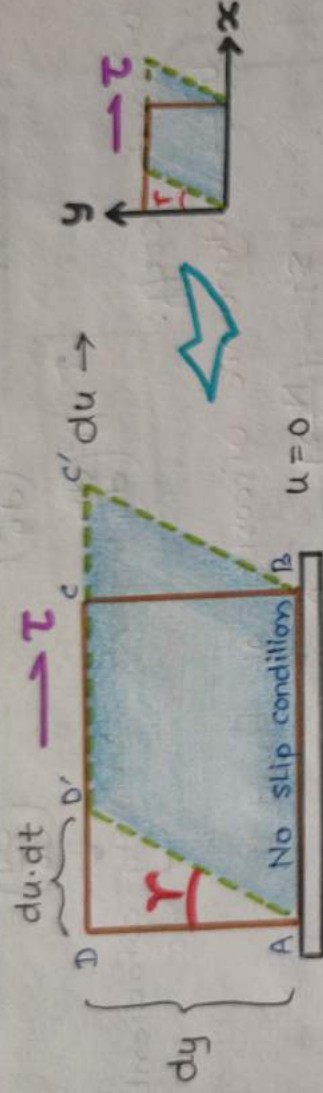
\rightarrow dynamic viscosity
 \rightarrow density of fluid

this is - ability of any fluid to diffuse a disturbance in molecular momentum



Viscosity \rightarrow Resistance to flow of a fluid
 Reason \rightarrow Cohesive forces C-F liquid $>$ C-F air
 Intermolecular momentum exchange

Fluid element deformation



Velocity of fluid at solid boundary = 0
 velocity of fluid layer at dy altitude = du
 in time interval $\rightarrow dt$
 displacement of upper fluid layer = $du \cdot dt$
 Now

(γ = shear strain) \rightarrow angle travelled through deformation

as; $\tan \gamma = \frac{DD'}{DA} = \frac{du \cdot dt}{dy} = \left(\frac{du}{dy} \right) dt$

if $\gamma \rightarrow$ very small then $\tan \gamma = \gamma$

$\gamma = \left(\frac{du}{dy} \right) \cdot dt$

$\frac{\gamma}{dt} = \left(\frac{du}{dy} \right)$

$\dot{\gamma} = \left(\frac{du}{dy} \right)$

$\dot{\gamma}$ = rate of shear strain
 shear strain per unit time
 $\left(\frac{du}{dy} \right) =$ Velocity gradient

Shear strain rate = Velocity gradient

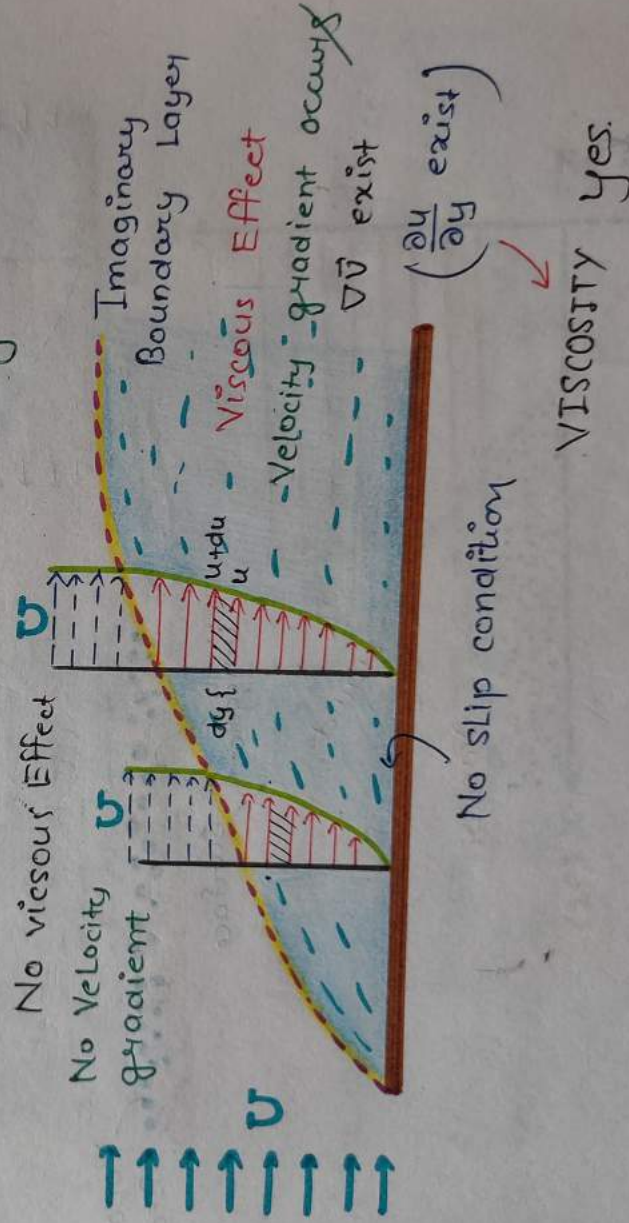
VISCOUS EFFECT

Following NO SLIP Boundary conditions, the first layer of fluid molecules, adhering to the plate, feels the presence of the wall directly and tends to stick to the wall with Relative velocity equal to Zero ($V_{relative} = 0$)

Layers above it do not feel presence of wall directly rather it feels the effect of the wall implicitly by virtue of some fluid property

enables the effect of MOMENTUM DISTURBANCE imposed by the wall to propagate from the wall, adjacent layer to the furthest outer layers

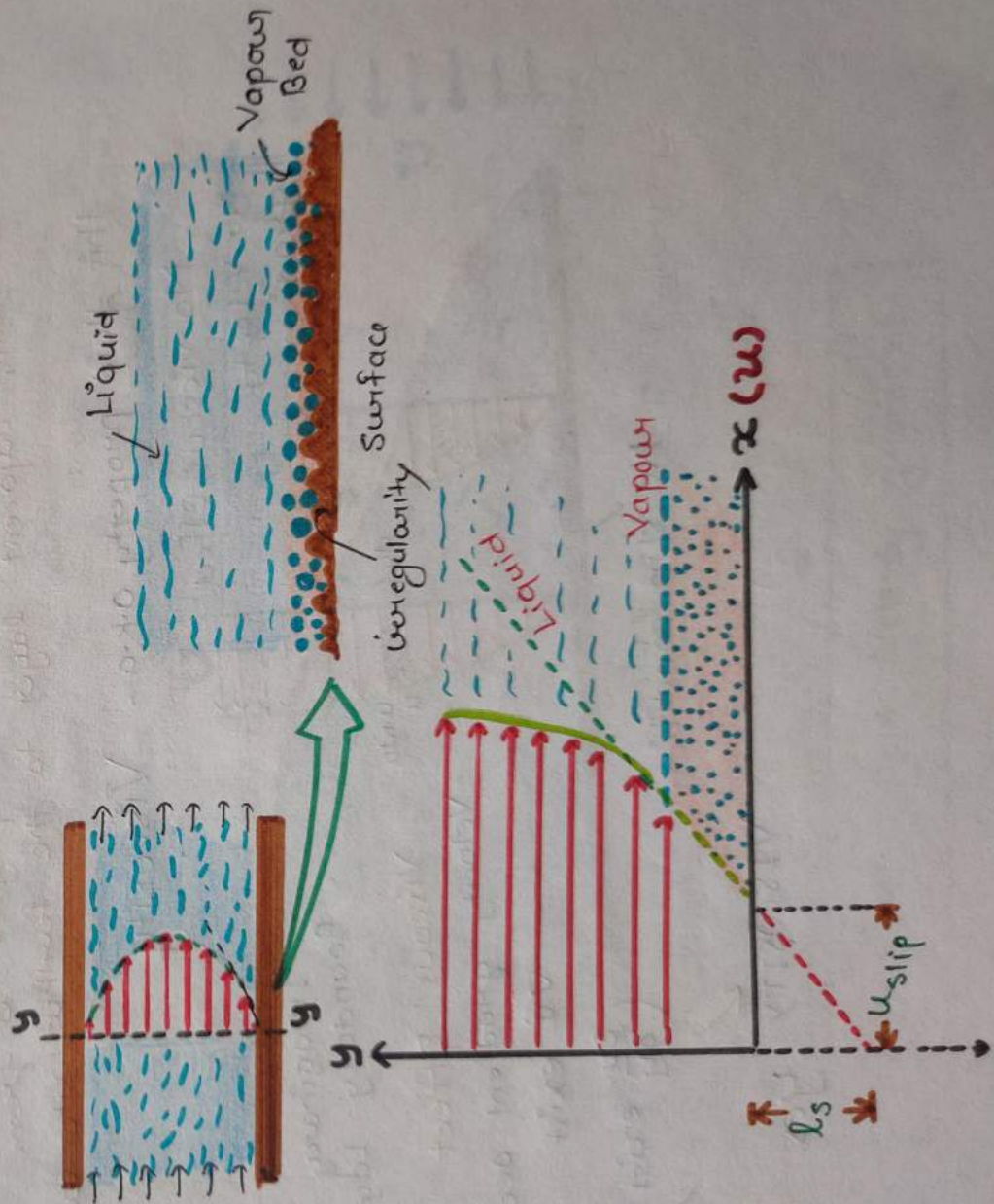
this fluid property a.k.a - Viscosity



Vapour \rightarrow acts like shield \rightarrow preventing the fluid from being directly exposed to the irregularities of the channel surface
 liquid not feel the presence of wall directly
 [liquid element sailing over intervening vapour bed.]
 APPARENT - SLIP happens now

$$u_{slip} = l_s \left. \frac{\partial u}{\partial y} \right|_{\text{interface}}$$

$l_s \rightarrow$ slip length (Height of Vapour bed)
 $u_{slip} \rightarrow$ slip velocity of liquid (sailing over vapour)



Second theory

No slip boundary condition arises due to microscopic boundary roughness, since fluid elements may get trapped within the surface asperities, if the fluid is liquid, it may not be possible for the molecules to escape from trapping because of compact molecular packing.

otherwise smooth boundary allows to slip

But recent study have demonstrated that the intuitive assumption of "NO SLIP at Boundary" can fail greatly not only when surface is smooth But also when surface is sufficiently rough.

Don't use

True slip

Rather use

Apparent slip

attempt to resolve apparent anomaly of "reduced" fluid friction in the presence of rough surface elements under specific conditions.

Roughness impedes motion of adhering fluid

A thin layer of Vapour formed on Roughness (nanometer scale) spontaneously by

because of Hydrophobic surface
Narrowness of the [surface don't easily get wet]
confinement.

First Theory →

molecules of the fluid next to solid surface, are absorbed onto surface for a short period of time, and are then desorbed and ejected into the fluid. This process slows down the fluid and renders the tangential component of the fluid velocity equal to the corresponding component of boundary velocity.

remains valid only if

Fluid adjacent to solid boundary is in the state of

THERMODYNAMIC EQUILIBRIUM

infinitely large No. of collisions needed b/w fluid molecules and the solid surface

Not possible for **Heavy** or **Less dense medium**

may result in a "SLIP" between fluid and solid boundary in small channels. ($\lambda \approx$ channel dimensions)

this event may be more aggravated by the

Presence of local gradient of

Temperature (∇T)

thermophoresis

density ($\nabla \rho$)

diffusiophoresis

Molecules tending to slip on the wall experience a Net driving force due to

∇T

$\nabla \rho$

Local Temp. gradient

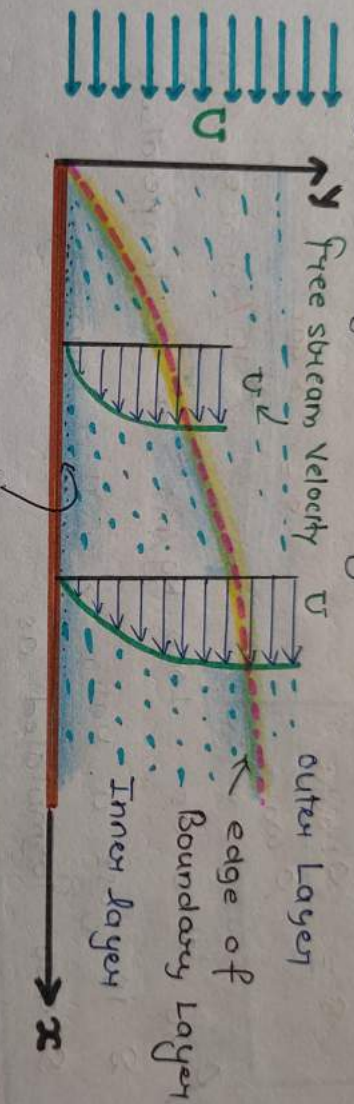
Local density gradient

Local density gradient

VISCOSITY

The No-SLIP Boundary Condition

Consider a flat plate of large width being kept in a "free stream" flow. So that the fluid just before encountering with the plate is having a uniform velocity U



No slip boundary condition

First set of fluid molecules comes in contact with the plate, These molecules tend to stick to the solid plate.

No Relative Tangential velocity component b/w fluid and solid Boundary at their contact points, this concept used in common fluid dynamics engineering problems.

NOTE

Physical origin of NO-SLIP Boundary Condition over a solid Boundary has Not been established with certainty. A matter of strong debate in Research Comm.

also known as **Relative density**

Specific gravity shown by symbol **S**

measurement of **S** to find any substance lighter or heavier with respect to reference substance

$$S = \frac{\text{density of substance}}{\text{density reference subst.}}$$

Liquid gas

H_2O H_2/air

$$S = \frac{\rho_{\text{Liquid}}}{\rho_{\text{water}}}$$

$$S = \frac{\rho_{\text{gas}}}{\rho_{H_2/air \text{ at Room Temp.}}}$$

ρ_{H_2O} taken as 1000 kg/m³ at 4°C temperature

S is just a ratio \rightarrow unitless quantity

S also formulated as

$$S = \frac{m_L}{m_{H_2O}} = \frac{w_L}{w_{H_2O}}$$

$$S = \frac{m_{\text{substance}}}{m_{\text{reference}}} = \frac{w_{\text{substance}}}{w_{\text{reference}}}$$

Mercury (Hg) = 13.6

S

Apparent specific gravity (**ASg**)

$$S_{\text{app}} = \frac{W_{\text{liquid}}}{W_{H_2O}} \Bigg| \text{Volume Constant}$$

API gravity measure of how heavy or light petroleum liquid is as compared to water

$$API = \frac{141.5}{S} - 131.5$$

API > 10° Lighter than H_2O
API < 10° Heavier than H_2O

mass (m) is amount of the substance
 Unit S.I. kg
 C.G.S. g
 1 kg = 10³ g

mass = density \times volume

cartesian = $\int \rho \cdot dv = \iiint \rho \, dx \, dy \, dz$

m. cylindrical = $\int \rho \cdot dv = \iiint \rho \, r \, dr \, d\theta \, dz$

spherical = $\int \rho \cdot dv = \iiint \rho \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

Weight (W) is amount of force any substance is pulled by earths (or any planet)
 it can vary
 Unit - S.I. - Newton N

Weight = mass \times acceleration due to gravity
 Mks - 1 kgf ($\approx 9.8 \, N$)

W cartesian = $mg = \int \rho g \, dv = \iiint \rho g \, dx \, dy \, dz$
 cylindrical = $mg = \int \rho g \, dv = \iiint \rho g \, r \, dr \, d\theta \, dz$
 spherical = $mg = \int \rho g \, dv = \iiint \rho g \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

Specific Volume

Shown by v or V
 is volume per unit mass

$$v = \frac{V}{m} = \frac{1}{\rho}$$

Unit m^3/kg (S.I.)
 cm^3/gm (C.G.S.)

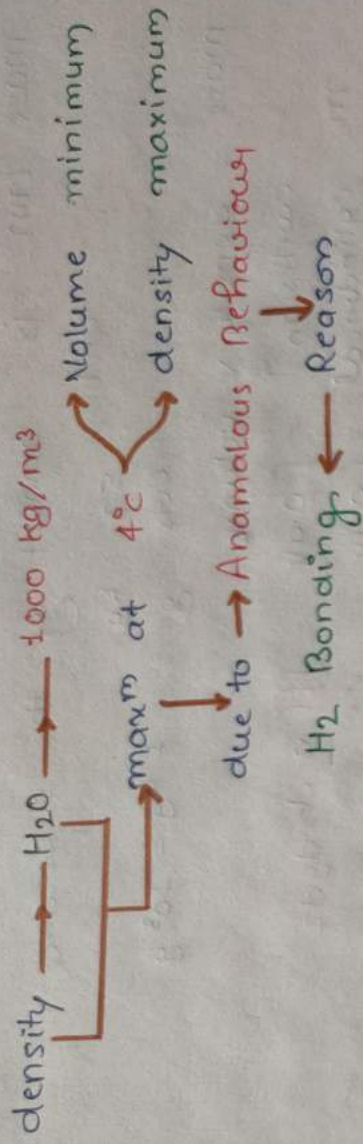
Specific Properties

Specific weight

Shown by w
 is weight per unit Volume

$$w = \frac{W}{V} = \rho g$$

Unit. N/m^3 (S.I.)
 $dyne/cm^3$ (C.G.S.)

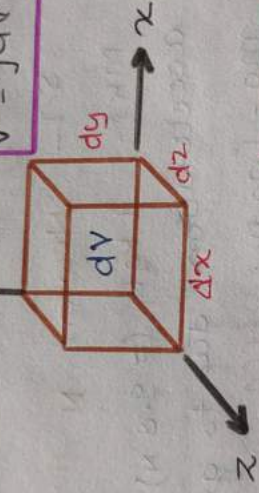


Volume is quantity of space occupied

Formulated in different coordinate systems

Cartesian Coordinate system

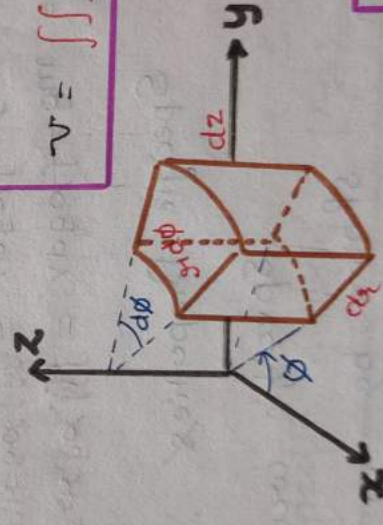
$$V = \int dV = \iiint dx dy dz$$



Cylindrical coordinate system

$$dV = r dr d\phi dz$$

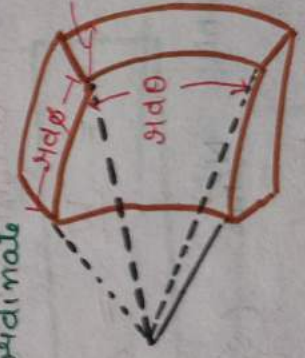
$$V = \iiint r dr d\phi dz$$



Spherical coordinate system

$$dV = r^2 dr d\theta d\phi$$

$$V = \iiint r^2 dr d\theta d\phi$$



if $\delta V \rightarrow \text{Too Large}$

there will be density variation within volume then average density must differ from the actual density at the center of sampling volume

Spatial variation of density

$$\rho = \rho(x, y, z, t)$$

Macroscopic Uncertainty

Therefore $\delta V \rightarrow$ Not Too Large, Not so tiny must contain around million molecules, density at a point in space is specified by

$$\rho = \lim_{\delta V \rightarrow \delta V^*} \left(\frac{\delta m}{\delta V} \right)$$

$\rho \rightarrow$ continuous and continuously differentiable function can be termed as

$$\rho = \rho(x, y, z, t)$$

similarly, other parameters such as - Velocity also show continuous & differentiable functionality.

$$u = u(x, y, z, t) \quad v = v(x, y, z, t) \quad w = w(x, y, z, t)$$

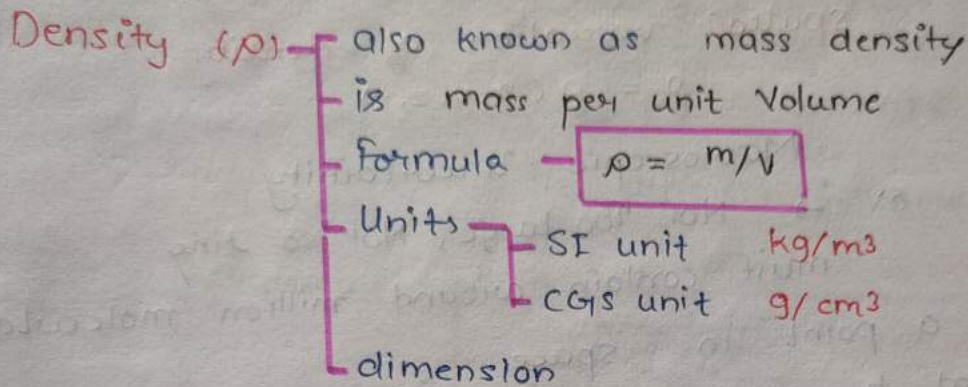
$$\text{Velocity vector } \vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

NON-VALIDATION

- ✓ *Maxified gases* (when pressure $\rightarrow 0$)
No continuum concept can sustain
- ✓ microfluidics application
- ✓ flow devices at micro & nano scale

PROPERTIES OF FLUIDS

We will study, here, some mathematical and conceptual details of fluidic properties.



detailed discussion

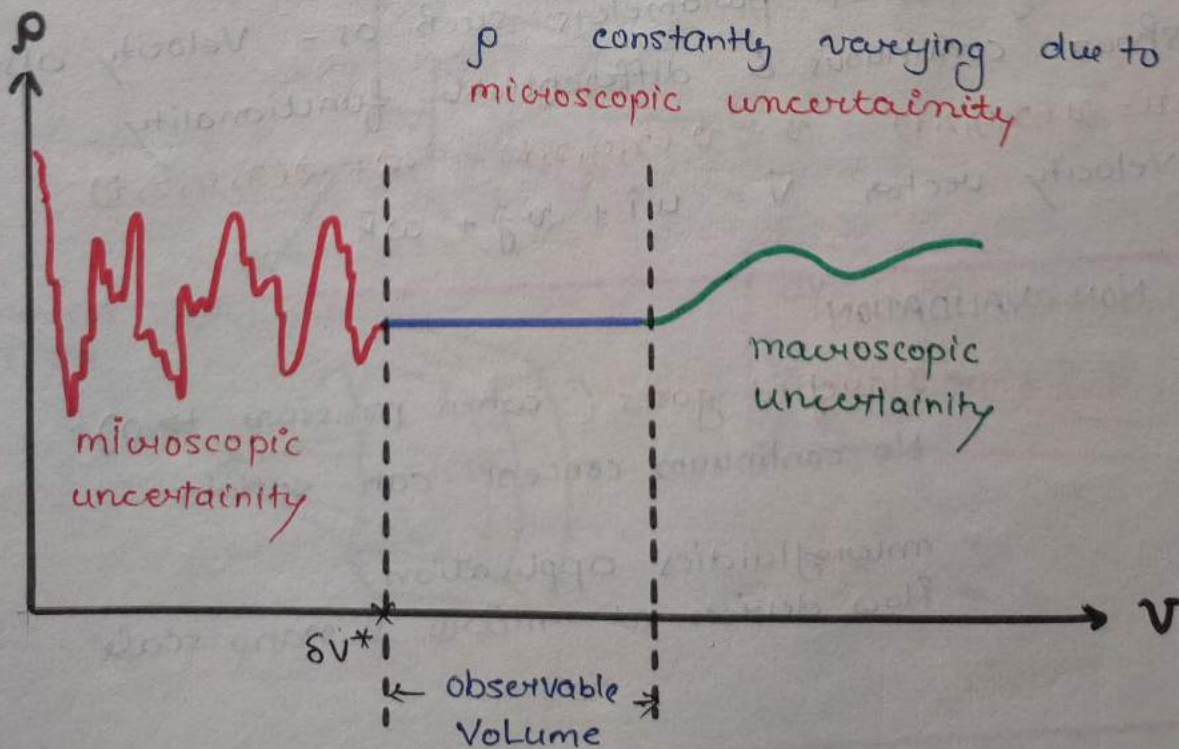
under consideration of continuum

density $\rho = \frac{\delta m}{\delta V}$

sampling mass

sampling volume

$\delta V \rightarrow$ very small random motion dominant. then



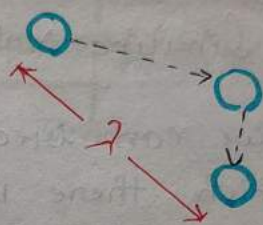
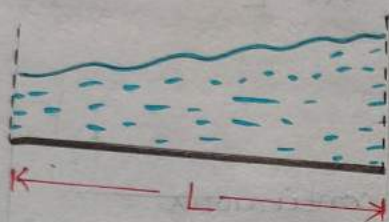
Knudsen Number

$$(\lambda/L)$$

Indicator of degree of rarefaction of system
determines the extent of deviation from a possible continuum behaviour

$$Kn = \frac{\lambda}{L}$$

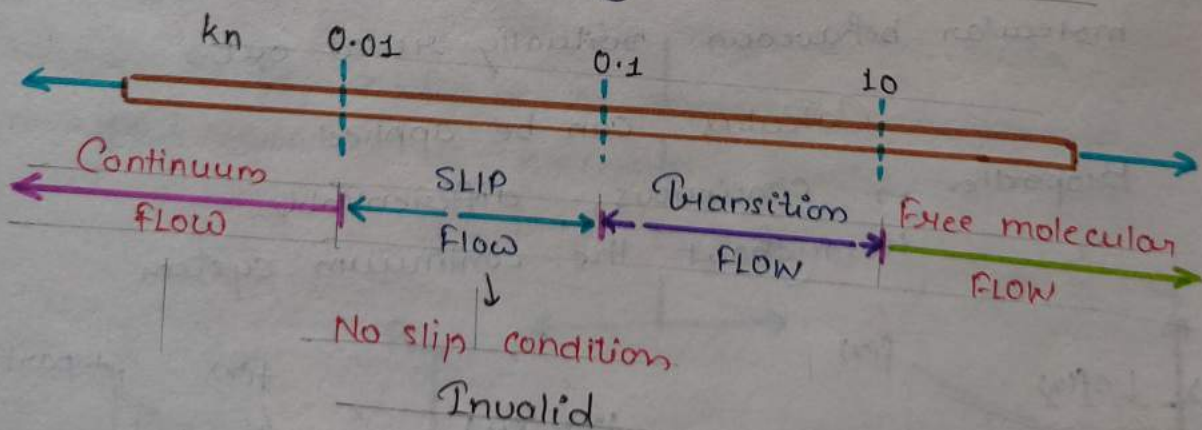
mean free path
characteristic length



Average distance travelled b/w to successive collisions
↓
 λ

Length over which we have to measure the property (Length of observation)

Knudsen Number ranging flow



Continuum \rightarrow Continuous Medium

Too small

Too large

Lot of uncertainty here because of larger fluctuation of the values of our property

$$\lambda > L$$

we shall not be able to capture the local variation of properties.

$$\lambda \lll L$$

Limiting Volume

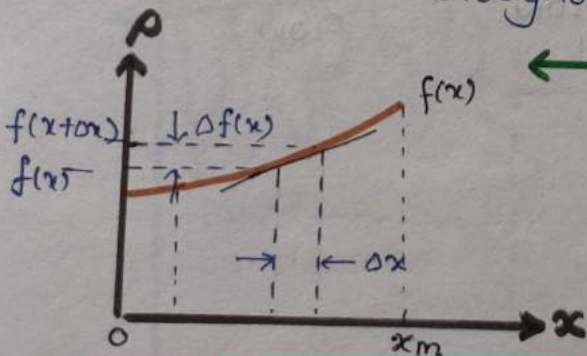
above which we can treat as continuous medium in which there is no significant uncertainty in prediction of the averaged behaviour of molecules in the system.

variation of the property within this medium is smooth enough

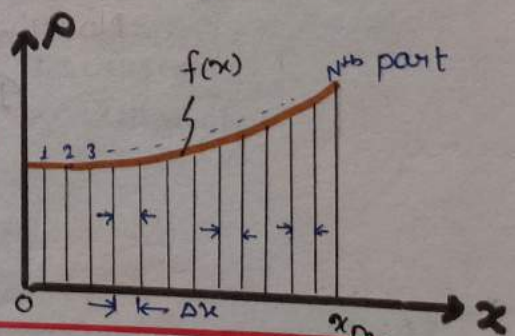
randomness due to uncertainties in the molecular behaviour virtually ruled out.

Calculus can be applied

Properties \rightarrow continuous, differentiable throughout the continuum system



$$\Delta x = \left(\frac{x_m}{N} \right)$$



$$\frac{df}{dx} = \lim_{\Delta x \rightarrow 0} \left\{ \frac{f(x+\Delta x) - f(x)}{\Delta x} \right\}$$

$$\int_0^{x_m} f(x) dx = \lim_{\Delta x \rightarrow 0} \left[\sum_{i=1}^N f_i(x) \Delta x \right]$$



Continuum Hypothesis



fluid appears **continuous**

Made of molecules

Empty space b/w molecules

Quantify fluid properties

On microscopic length scale (μm)

NOT on molecular length scale (nm)

Ignore molecular details/spaces

Assume fluid as **continuous medium**

Continuous function to quantify fluid properties

Need to make continuum Hypothesis

Artificial model for convenience

Do not consider

Motion of individual molecule

forces on individual molecule

Involves fewer and fewer details

Easy to compute the data

Behaviour of billions of molecules summarized in few continuous functions

density

pressure

velocity

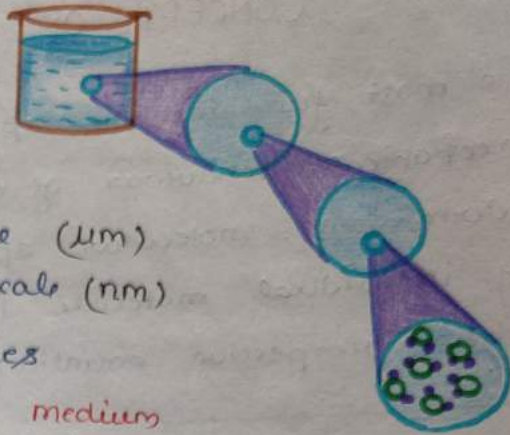
Temperature

concentration

Lost is not important

↳ details of molecular arrangement and its motion

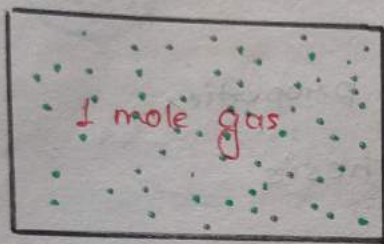
↳ Brownian motion type



MACROSCOPIC and MICROSCOPIC point of View

CONCEPT OF CONTINUUM

the most fundamental approach to analyse the mechanical behaviour of a fluidic system may be a deterministic molecular approach in which dynamics of individual molecule is investigated by writing their respective equations of motions.



6.023×10^{23} molecules

6.023×10^{23} equations of motion

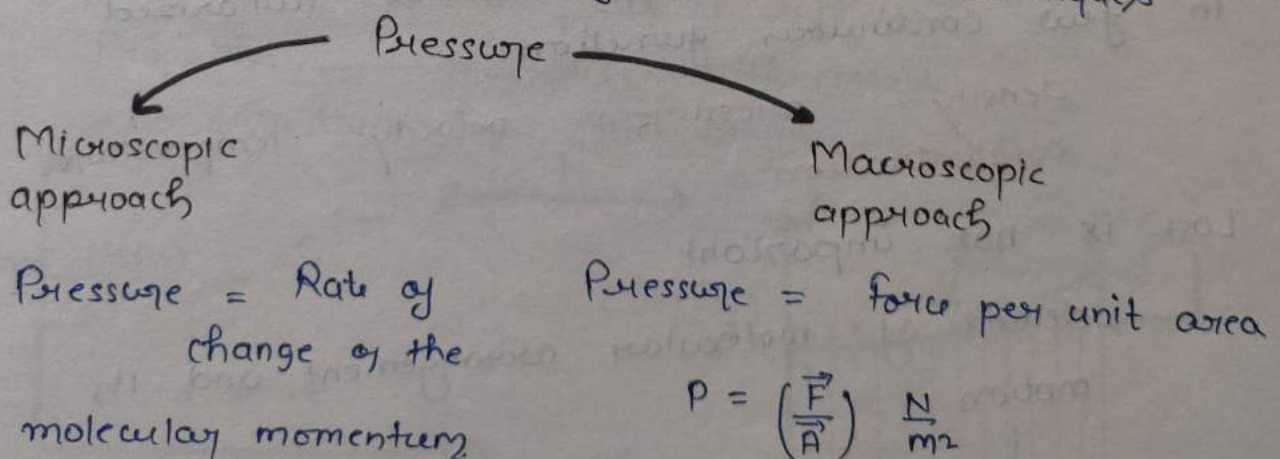
Very hard to compute today even to search for an approach which can reduce No. of equations

MICROSCOPIC approach

statistically averaged behaviour of many molecules

MACROSCOPIC approach

gross effect of many molecules that can be captured by our sensing measurement techniques



Solid

Fluid (Liquid)

: can resist applied shear stress τ by deforming

: can't resist applied shear stress (τ)

: No continuous deformation

: Continuous deformation

(γ Not fixed)

: Hooke's Law of elasticity

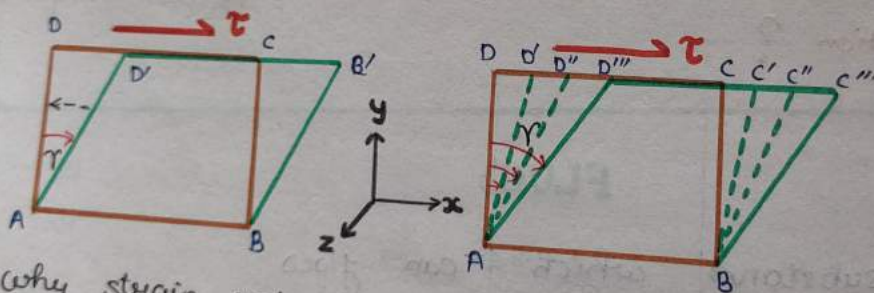
: Newton's Law of viscosity

stress \propto strain

stress \propto strain rate

: can restore its original shape after τ removal

Can't restore when τ removal



why strain rate not strain?

as we know, fluid element tends to deform continuously. its angle of deformation (shear strain) continuously changes with time. Therefore to measure the deformation we use **rate of strain** (change of strain along with time)

Broad Perspective to FLUID MECHANICS

Fluid mechanics deals with the behaviour of fluid (liquids and gases) in rest or in motion.

Some questions can be tackled with Fluid Mechanics

- : How does a rocket go up?
- : How do insects fly?
- : How does blood flow through arteries and veins?
- : How does a human heart act like a pump?
- : How are ocean currents formed?
- : How should one design the shape of a car to minimise the wind resistance against its motion?

FLUID

any substance which can flow

[finite mass
occupy space
Tangible]

[relative change of position
of particles with respect
to time]

Liquid
Gas

Viscosity ✓
Shape / Volume indefinite
Flow under own weight ✓
Continuous deformation ✓

Solids

Viscosity ✗
Flow ✗
Flow under own weight ✗
Continuous deformation ✗